

Critical phenomenon of granular flow on a conveyor belt

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The relationship between the granular wafer movement on a two-dimensional conveyor belt and the size of the exit together with the velocity of the conveyor belt has been studied in the experiment. The result shows that there is a critical speed v_c for the granular flow when the exit width d is fixed (where $d=R/D$, D being the diameter of a granular wafers). When $v < v_c$, the outflow rate Q increases linearly with the speed v and the flow rate $Q = \rho v R$. The turning point of the Q - v curve occurs at the speed v_c . The critical speed v_c is dependent on the exit width d . When $v > v_c$, the flow rate Q is described as $Q = C \rho v^\beta (d - k)^{3/2}$. These are the effects of the interaction among the granular wafers and the change of the states of the granular flow due to the changing of the speed or the exit width d .

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INTRODUCTION

Granular matter exists extensively in nature and in our daily lives. It appears in desert, avalanche, landslide, and floating ice as well as in traffic flow and engineering chemistry. The regularity of the motion of the discrete system of this kind is quite complicated. As a result, the understanding of these properties and its behavior is rather insufficient. It has been a fascinating phenomenon of much interest to physicists in recent years [1–6]. The behavior of granular flow in a funnel has been studied early where the mass-flow rate is independent of the height of granular in the funnel but related to the exit width of the funnel. The flow rate can be described [5,7] as $Q = C \rho_b \sqrt{g} (D_0 - kD)^{5/2}$, where D_0 is the opening size of the funnel, ρ_b is the density of the granules, g is the acceleration due to gravity, D is the diameter of the granules, k and c are constants, respectively. For the two-dimensional flow, the equation can be overwritten as $Q = C \rho_b \sqrt{g} (D_0 - kD)^{3/2}$. To and co-workers [8] recently studied the jamming of granular flow in the two-dimensional hopper. The result showed that the jamming probability is a function of the funnel opening. The jamming probability approaches to 1 when the opening size is less than four times of the diameter of the granules, i.e., the jamming happens when $D_0 \leq 4D$. In this paper, we will report our studies on the relation between the flow rate Q and speed of the conveyor belt when the granular wafers passes through the bottleneck. Our results may be helpful in the understanding of the transition from free flow to jamming in the traffic flow [9,10].

EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. The copper disks of 6 mm in thickness and 16 mm in diameter are put on the two-dimensional conveyor belt with the continuously adjustable speed from 0.10 to 2.75 m/s. On the twain sides are

placed a pair of baffles made of aluminum alloy 300 mm apart. Monolayer granular wafers are added from the inlet. On the other end, the granular wafers flow out of the exit where the exit is like a bottleneck. The baffle walls make an angle $\phi = 90^\circ$ with the direction of the moving velocity of the granular wafers. The exit width R can be adjusted continuously, which is kept less than 150 mm ensuring the granular wafers to be suffocated by the bottleneck. A weighting sensor with sensitivity of 1.0 g and recording rate of 0.02 s is placed under the opening exit. The experimental data are transmitted to computer instantaneously, i.e., the granular mass $M(t)$ passing through the opening, the function of time can be recorded. The outflow rate $Q = dM/dt$ was measured for each exit width R . We defined $d = R/D$, where D is the diameter of a granular wafer and d is the exit width in terms of the number of the disk diameters. In our case, the jamming probabilities are close to 1 when d is less than 3.0, we will report only the results with the opening d larger than 3.0.

EXPERIMENTAL RESULTS

We measured the outflow rates for eight different exit sizes d from 3.5 to 8 and the speeds of the conveyor belt were changed from 0.16 to 1.16 m/s. Figure 2 shows that the relation curves between the flow rates and the speeds of the conveyor belt for different exit sizes. It can be derived from Fig. 2 that the granular flow rate rises as a result of the increase of the velocity of the conveyor belt and the relation-

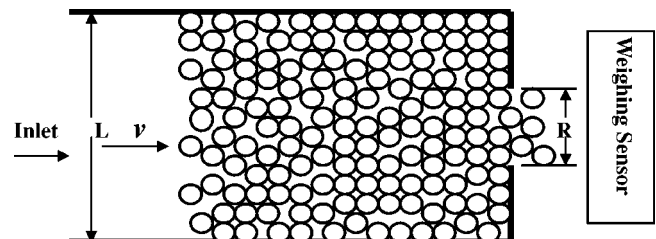


FIG. 1. Schematic diagram of the experiment setup.

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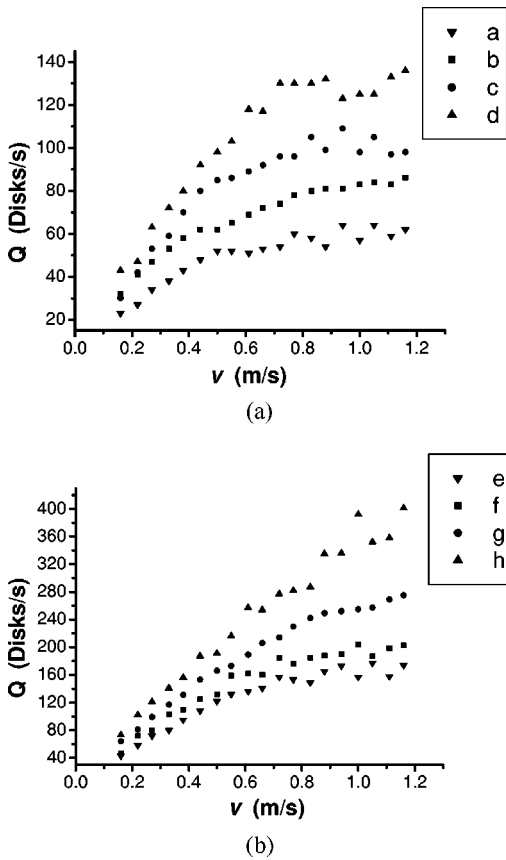


FIG. 2. The flow rate Q relation to the speed of the conveyor belt v in different opening d . In Fig. 2(a), a to d correspond to different openings 3.5, 4.0, 4.5, and 5.0. In Fig. 2(b), e to h to different openings 5.5, 6.0, 7.0, and 8.0.

ship between the two appears to be nonsimple. The outflow rates Q vary approximately linearly with the speed of the conveyor belt when its speed is low. There is a transition of the flow rate Q when the speed of the conveyor belt v reaches the critical speed v_c . For example, at a fixed exit width $d=3.5$, the relation between the outflow rate and the speed of the conveyor belt is shown in Fig. 3. One transition

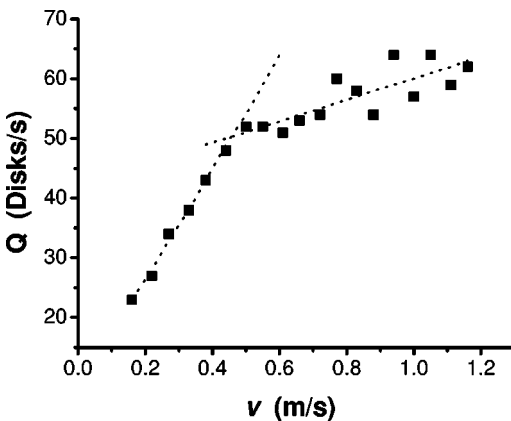


FIG. 3. The flow rate Q relation to the speed of the conveyor belt at a fixed opening $d=3.5$. The dotted line is the result of simulation.

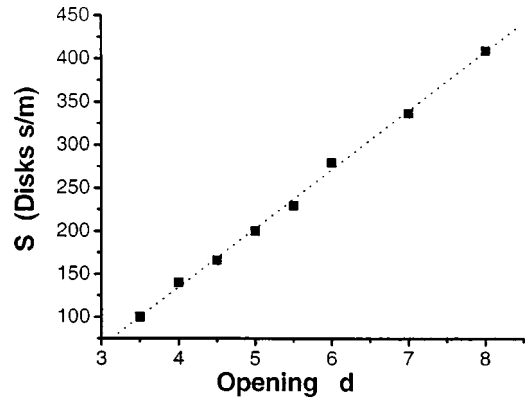


FIG. 4. The S relation to d when $v < v_c$, where $d \equiv R/D$ is the opening in units of disk diameter.

of the flow rate Q emerges when the velocity reaches the critical v_c . And it can be read that v_c equals 0.44 m/s. The Q vs v is linear when $v < 0.44$ m/s, the slope S equals to 94 (defining $S = \Delta Q / \Delta v$). When $v > 0.44$ m/s, the Q vs v is quite dispersive. For different openings, the S vs d is linear when $v < v_c$ as shown in Fig. 4.

The Q - v curve of each opening has its transition point v_c , respectively. The relation between the critical speed v_c and the exit width d for different openings is shown in Fig. 5. It shows that the v_c is generally linear with the exit width d . In our condition $v_c \approx 0.125d$.

The driving force of the motion of the granules is friction. The speed of the motion of the granules is identical with the speed of the conveyor belt if the exit width is large enough and the flow rate $Q = \rho v R$. Where ρ is the density of the granules. Therefore, the flow rate Q is linear with the speed v when $v < v_c$. In Fig. 4, the S vs exit width d is linear when $v < v_c$. We can elicit the result that the $\rho = 4280$ disks/m² from the slope $(\Delta Q / \Delta v) / \Delta d$ (where $d = R/16$ refers to the width of the opening in terms of the disk diameters) and the equation $Q = \rho v R$. The fraction of the area occupied by granular wafers in the two dimension approximates to 85%. It is analogous to that of the disarray system in two dimension. When $v < v_c$, the density of the granules ρ stays invariable at different exit width. It shows that the flow rate Q is

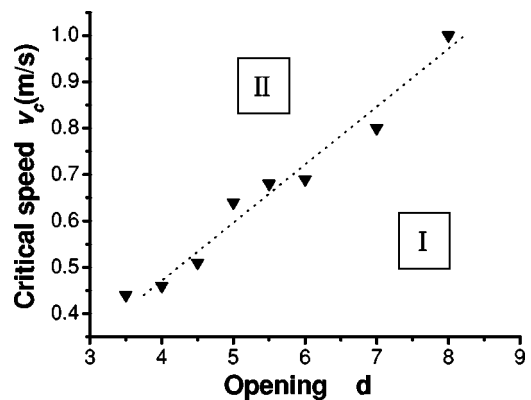


FIG. 5. The critical speed v_c relation to d , where $d \equiv R/D$ is the opening in units of disk diameter.

determined by the speed of the conveyor belt and the exit width.

However, when $\nu > \nu_c$, the flow rate Q is a nonlinear function with the exit width. The flow rate Q equals to $Q = C\rho\nu^\beta(d-k)^{3/2}$ approximately (where C is a constant, $k = 3.0$, $\beta \approx 1$). It can be derived from the Q - d curve when $\nu > \nu_c$ that β equals to 1 approximately. Furthermore, the flow rate has an obvious fluctuation in Fig. 3. This is due to the arching formed due to the baffle of the exit. In Fig. 5, it is obvious that ν_c is linear with respect to d . There is a critical line in Fig. 5. It is divided into two regions labeled region I and region II, respectively. Region I is called linear region, where the flow rate obeys the law $Q = \rho\nu R$ and region II is called nonlinear region, where the flow rate obeys $Q = C\rho\nu^\beta(d-k)^{3/2}$. This is because some disks cannot follow the same speed with the conveyor belt when the exit diminished to critical value d_c or the speed reached to the critical value ν_c . In fact, a transition from dilute flow to dense flow occurred because of the powerful interaction among the disks.

CONCLUSIONS

In the experiment the motion of monolayer granular wafers on the conveyor belt has been studied. The result shows that the rate of flow Q is influenced chiefly by the following

factors: Exit width R and the speed of the conveyor belt ν . There is a critical speed ν_c for the granular flow Q when the exit width R is fixed. The rate of flow Q increases linearly with the speed ν and the exit width R when $\nu < \nu_c$. It equals to $Q = \rho\nu R$ (where the ρ is the two-dimensional density of the granular disks). When $\nu > \nu_c$, the flow rate Q is a nonlinear function with the exit width. The flow rate Q equals to $Q = C\rho\nu^\beta(d-k)^{3/2}$ (where C is a constant, $d = R/16$ is the exit width in terms of the number of disk diameters, $k = 3.0$, $\beta \approx 1$). The transition of the Q - ν curve occurs at the speed of ν_c . The critical speed ν_c is dependent on the exit width d . In our experimental conditions, the critical speed ν_c increases linearly with the exit width d . Also, there is a critical exit width d_c when the speed ν of the conveyor belt is fixed.

The findings are helpful to the better understanding of the transition from free flow to jamming in traffic flow and the motion of other discrete matters.

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